Chapter 12: Vectors and Geometry in Space

12.1: Intro to R^3 (Video)

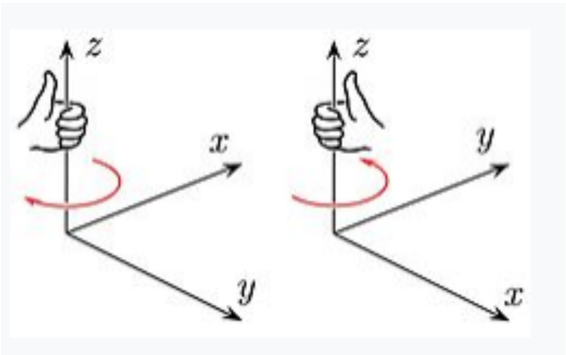
R: x

R^2: (x,y)

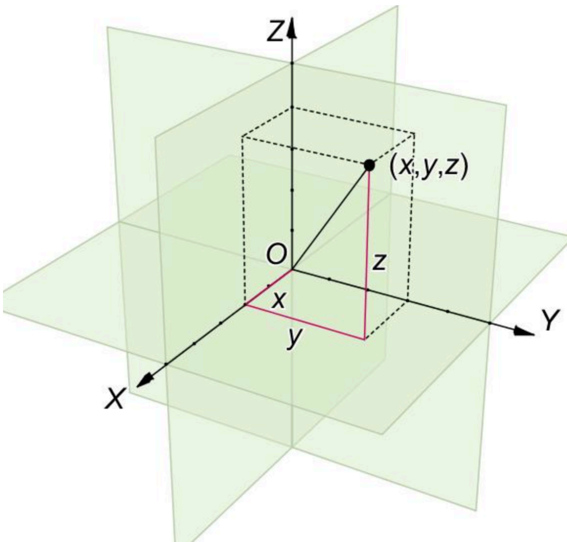
R^3: (x,y,z)

Ordered triple

Right Hand Orientation



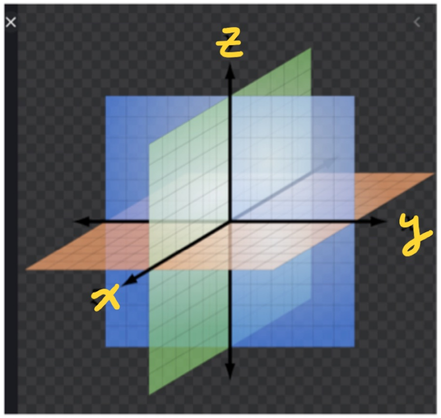
Plotting Points



Octants

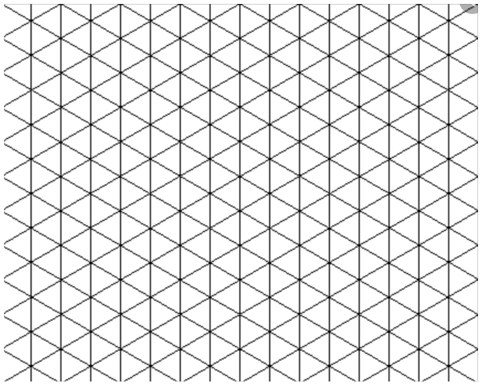
Coordinate planes:

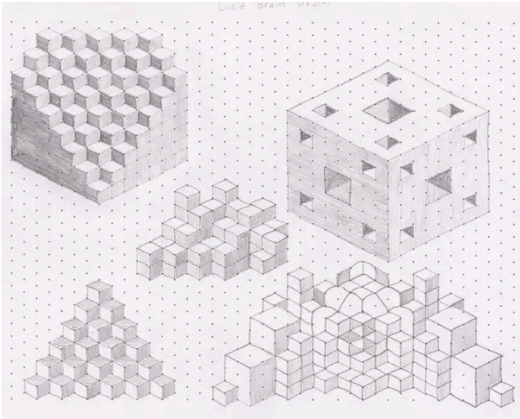
xy plane; z=0
yz plane; x=0
xz plane; y=0



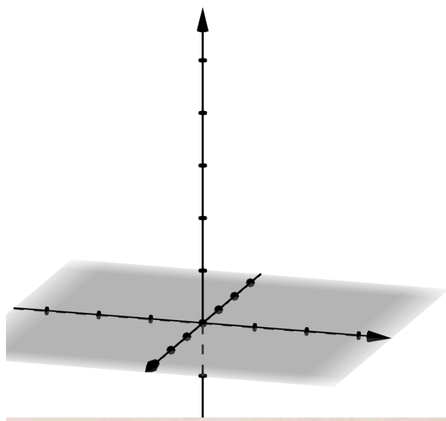
https://www.geogebra.org/m/mqGpuMUf

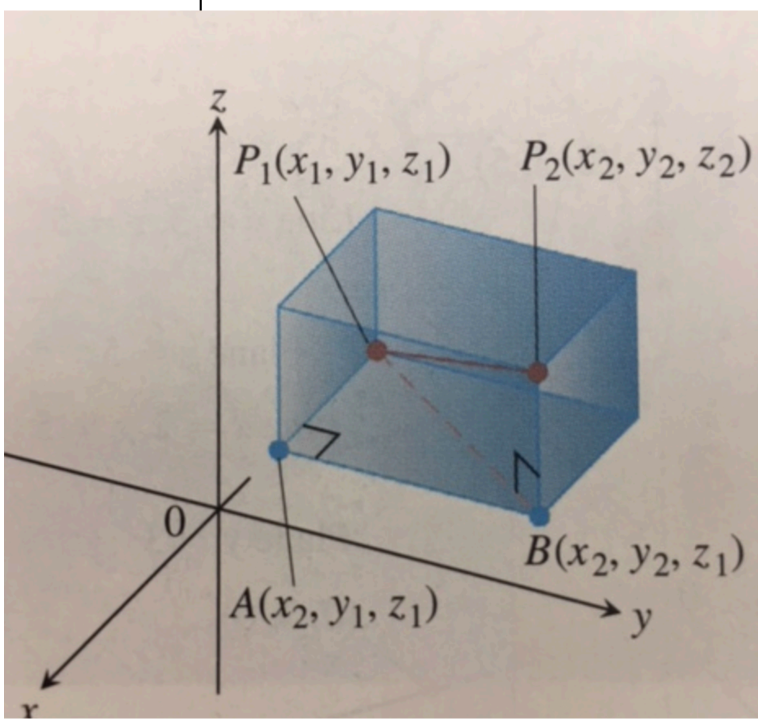
Isometric Grid Paper





Development of Distance Formula in R³.: Find the distance between points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$





Midpoint $P_1P_2 = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}, \right)$

Visualizing R^3:

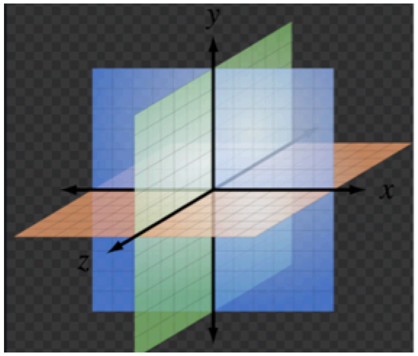
1. Determine whether each statement is true or false in $\mathbb{R}^3$.

(a) Two lines parallel to a third line are parallel.
(b) Two lines perpendicular to a third line are parallel.
(c) Two planes parallel to a third plane are parallel.
(d) Two planes perpendicular to a third plane are parallel.
(e) Two lines parallel to a plane are parallel.
(f) Two lines perpendicular to a plane are parallel.
(g) Two planes parallel to a line are parallel.
(h) Two planes perpendicular to a line are parallel.
(i) Two planes either intersect or are parallel.
(j) Two lines either intersect or are parallel.
(k) A plane and a line either intersect or are parallel.

Graphing in R^3 (12.1 cont'd and 12.6) (Video)

Sphere: The set of all points equidistant from a fixed point (h, k, l)

Example: Graph: $x^2 + y^2 + z^2 - 4x + 2y - 6z + 13 = 0$



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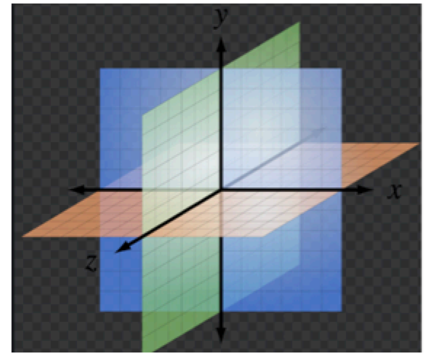
Plane: (more to come in 125)

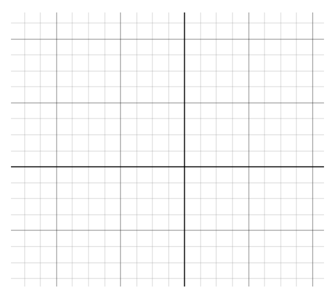


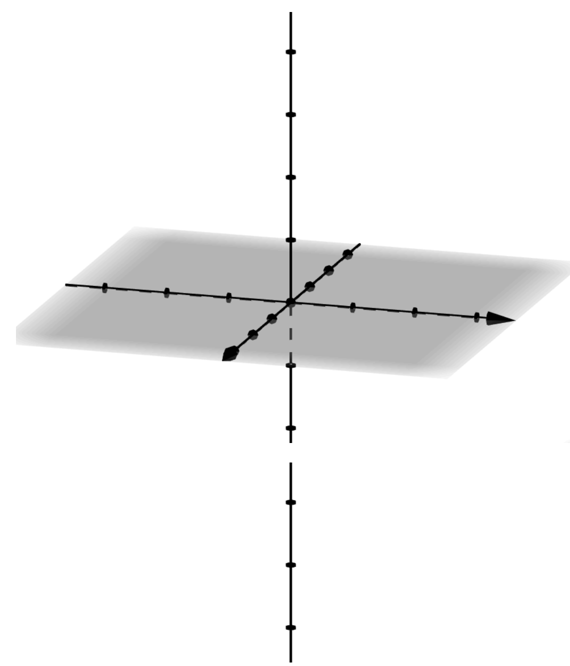
Cylinder: Not what you might think…

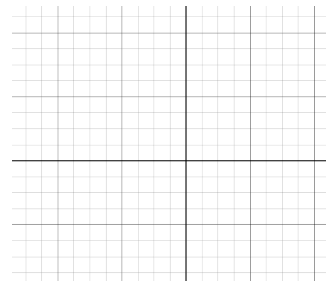
Cylinders

A **cylinder** is a surface that consists of all lines (called **rulings**) that are parallel to a given line and pass through a given plane curve.

Examples Cylinders:







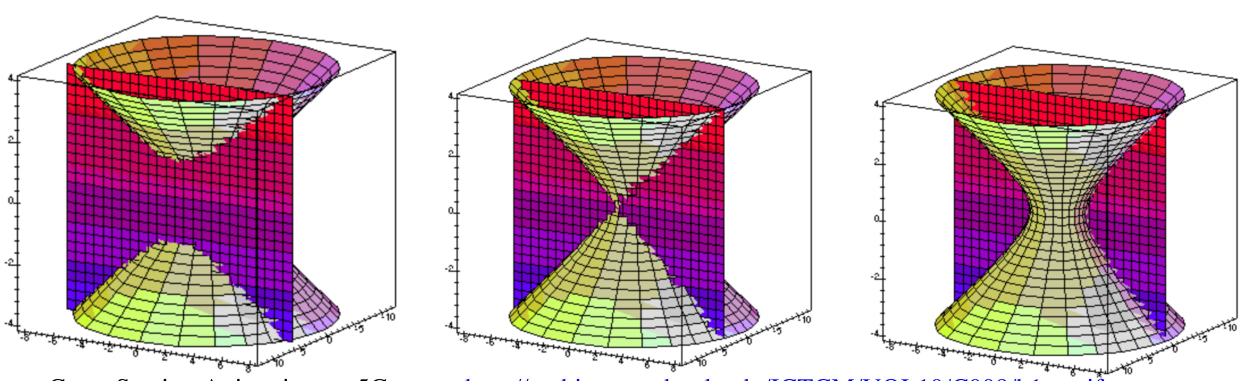
<u>General Second Degree Polynomials:</u>

In R^2

In R^3

Quadric Surfaces: (class)

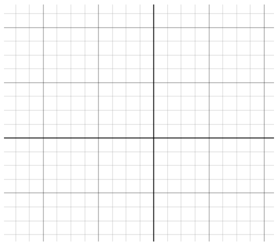
One technique in graphing involves considering traces x=k, y=k, z=k.

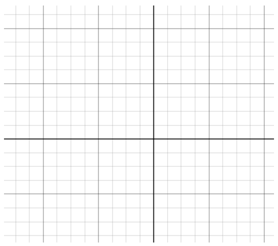


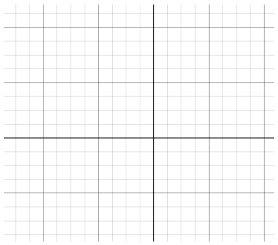
From Cross Section Animation on 5C page: http://archives.math.utk.edu/ICTCM/VOL10/C009/h1sv.gif

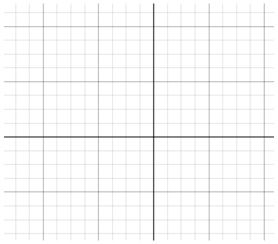
Example: Sketch $z = x^2 + \frac{y^2}{4}$

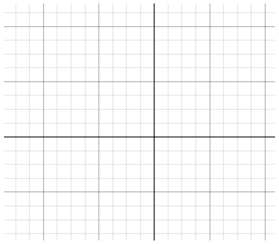
Consider Traces:

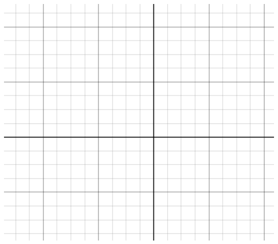


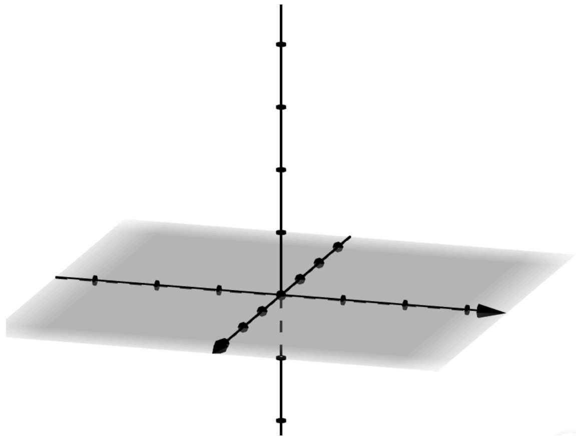




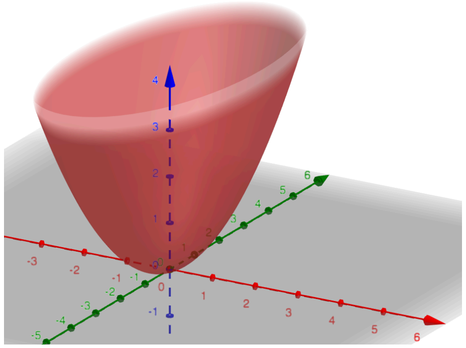


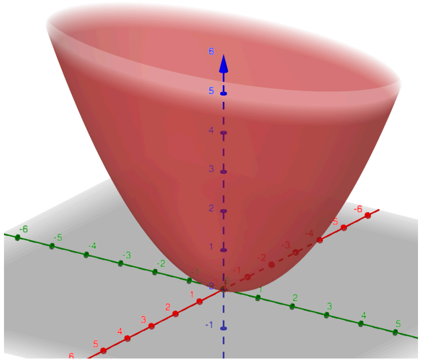


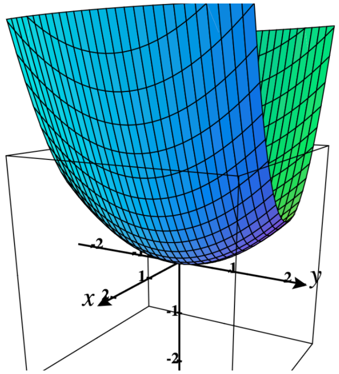


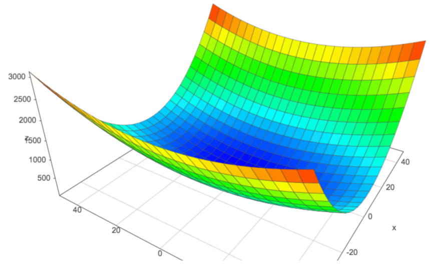


Graphing Software:



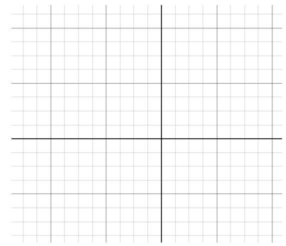


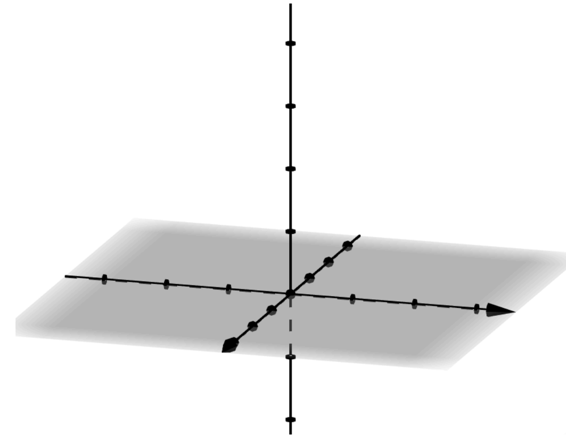




Elliptical Paraboloid: $\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ $\qquad \frac{y}{c} = \frac{x^2}{a^2} + \frac{z^2}{b^2}$ $\qquad \frac{x}{c} = \frac{z^2}{a^2} + \frac{y^2}{b^2}$

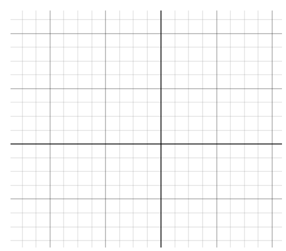
Example: Sketch: $y = \frac{x^2}{4} + \frac{z^2}{9}$; $\quad y = -\left(\frac{x^2}{4} + \frac{z^2}{9}\right)$

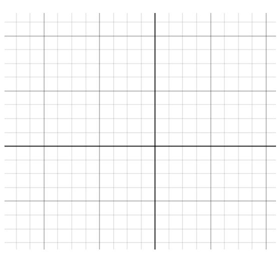


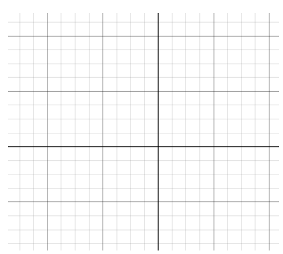


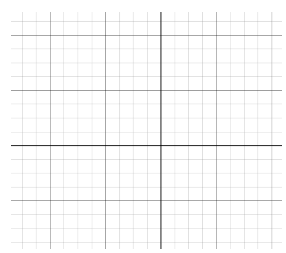
Hyperboloid of One Sheet: $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \qquad \frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \qquad \frac{-x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

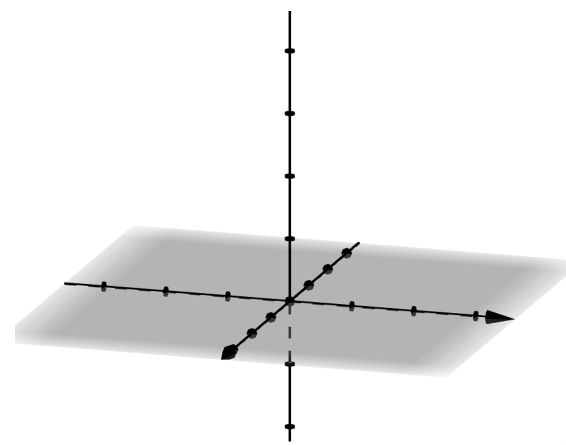
Example: Sketch $x^2 + \frac{y^2}{4} - \frac{z^2}{4} = 1$









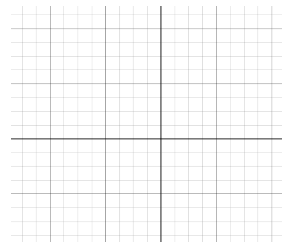


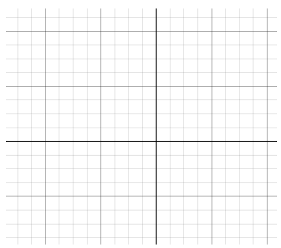
Show on geogebra with cross sections

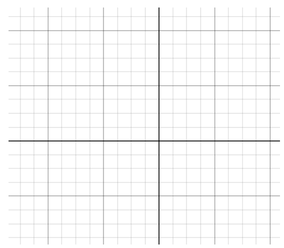
(Video)

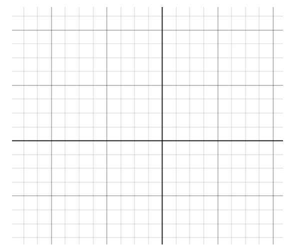
Hyperboloid of Two Sheets: $\frac{-x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ $\quad\frac{-x^2}{a^2}+\frac{y^2}{b^2}+\frac{-z^2}{c^2}=1$ $\quad\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$

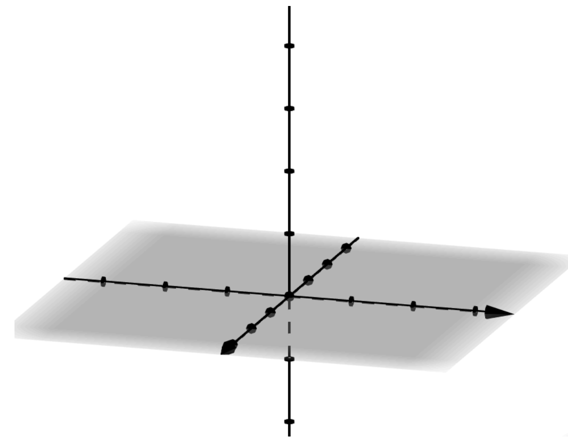
Example: Sketch $-x^2+\frac{y^2}{4}-\frac{z^2}{4}=1$





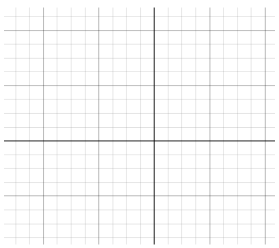


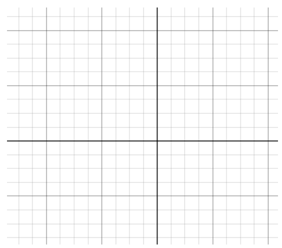


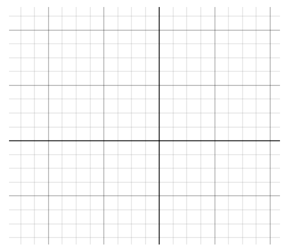


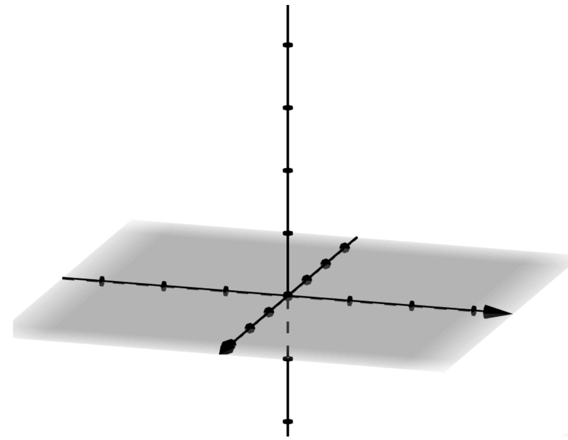
Hyperbolic Paraboloid: $\frac{z}{c}=\frac{x^2}{a^2}-\frac{y^2}{b^2}$ $\frac{y}{c}=\frac{x^2}{a^2}-\frac{z^2}{b^2}$ $\frac{x}{c}=\frac{z^2}{a^2}-\frac{y^2}{b^2}$

Example: Sketch $z=y^2-x^2$

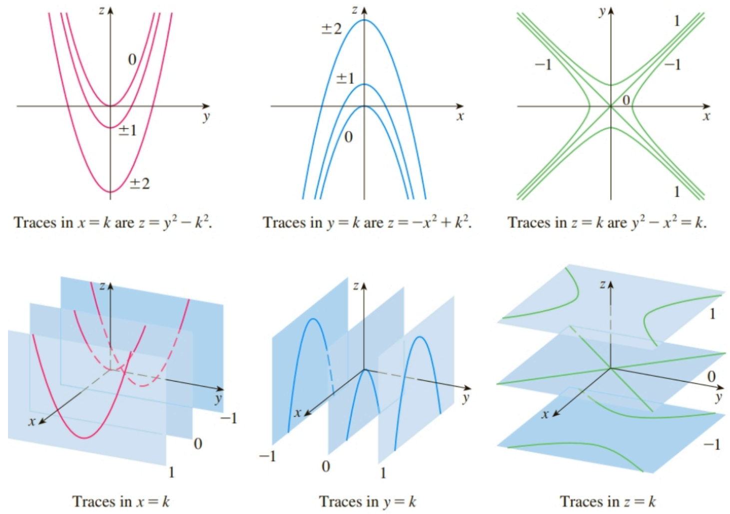








Example Hyperbolic Paraboloid Cont'd



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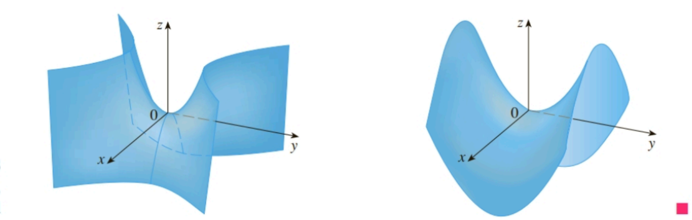
In Figure 8 we fit together the traces from Figure 7 to form the surface $z = y^2 - x^2$, a **hyperbolic paraboloid**. Notice that the shape of the surface near the origin resembles that of a saddle. This surface will be investigated further in Section 14.7 when we discuss saddle points.



See also in book:

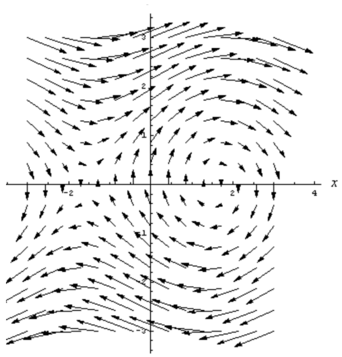
Ellipsoid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Cone: $\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ $\quad \frac{y^2}{c^2} = \frac{x^2}{a^2} + \frac{z^2}{b^2}$ $\quad \frac{x^2}{c^2} = \frac{z^2}{a^2} + \frac{y^2}{b^2}$ and half cone $\frac{z}{c} = \sqrt{\frac{x^2}{a^2} + \frac{y^2}{b^2}}$ …….

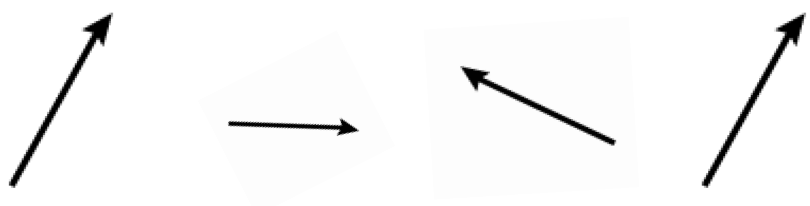
12.2 Vectors (Video)

A vector is a mathematical object that allows us to represent both _________________ and

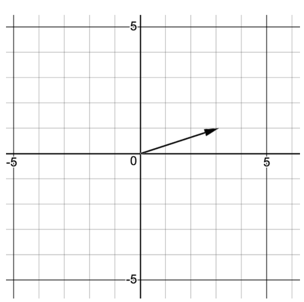
__________________. Vectors are often used physics and engineering.

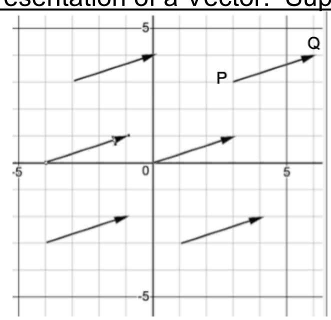


Geometric Representation of a vector:
Initial point, terminal point, notation, magnitude, direction, equal vectors

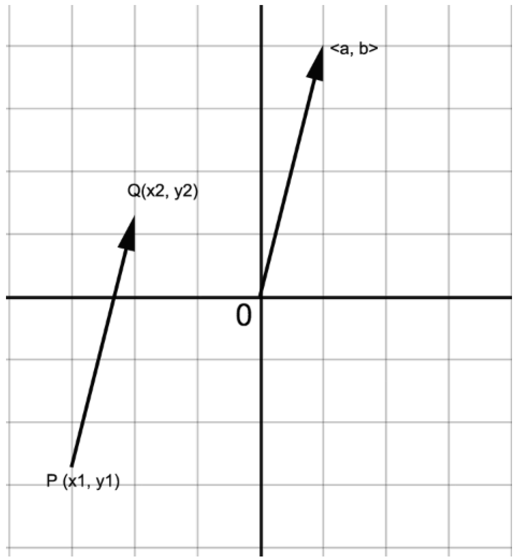


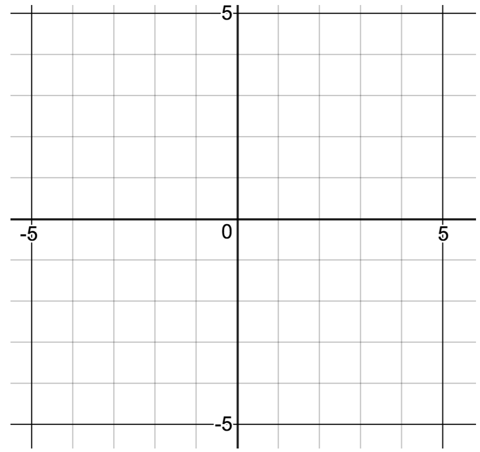
Algebraic (Component) Representation of a Vector: Superimpose a coordinate system





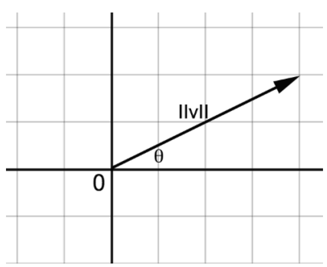
Computing the components of a vector $\vec{v}$ with representative $\overrightarrow{PQ}$ where the coordinates of points P and Q are given.



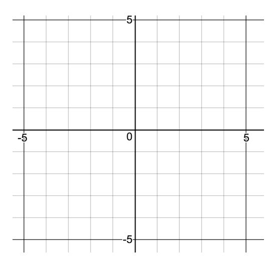


Converting: Magnitude/Direction to Component Form (if θ in standard position)

Given $\|\vec{v}\|$, θ find ___________________

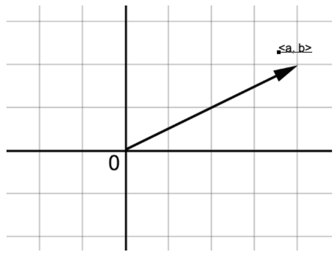


Example:



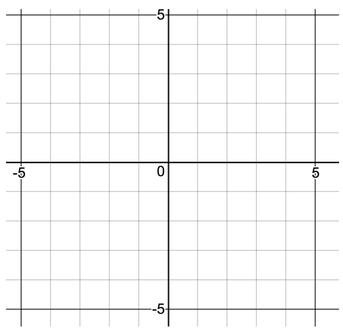
Converting: Component Form to Magnitude/Direction Form

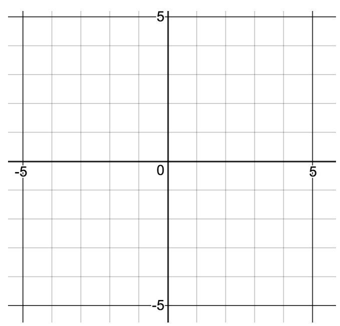
Given $\vec{v} = < a\ b >$ find ____________________

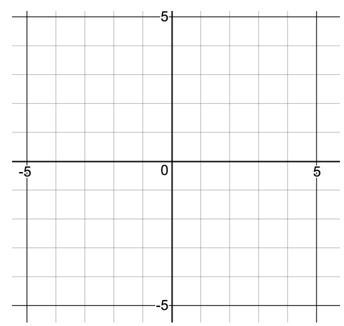


*** What quadrant is θ in?

Examples:



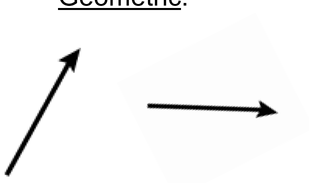




Vector Operations

Addition

Geometric:



Tip-to-Tail

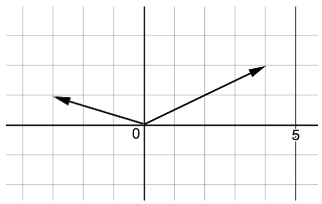
Parallelogram

Addition (cont'd)

Algebraic:

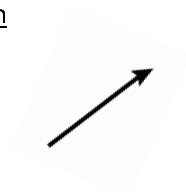
Suppose $\vec{v} = \langle v_1, v_2 \rangle$ and $\vec{w} = \langle w_1, w_2 \rangle$. The vector $\vec{v} + \vec{w}$ is defined by

$$\vec{v} + \vec{w} = \langle v_1 + w_1, v_2 + w_2 \rangle$$



Scalar Multiplication

Geometric:



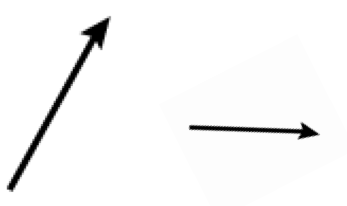
Algebraic

If k is a real number and $\vec{v} = \langle v_1, v_2 \rangle$, we define $k\vec{v}$ by

$$k\vec{v} = k \langle v_1, v_2 \rangle = \langle kv_1, kv_2 \rangle$$

Subtraction

Geometric: Adding Opposite Parallelogram



Algebraic:

Properties of Vectors If **a**, **b**, and **c** are vectors in V_n and c and d are scalars, then

1. $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$
2. $\mathbf{a} + (\mathbf{b} + \mathbf{c}) = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$
3. $\mathbf{a} + \mathbf{0} = \mathbf{a}$
4. $\mathbf{a} + (-\mathbf{a}) = \mathbf{0}$
5. $c(\mathbf{a} + \mathbf{b}) = c\mathbf{a} + c\mathbf{b}$
6. $(c + d)\mathbf{a} = c\mathbf{a} + d\mathbf{a}$
7. $(cd)\mathbf{a} = c(d\mathbf{a})$
8. $1\mathbf{a} = \mathbf{a}$

Zero Vector: ______________________

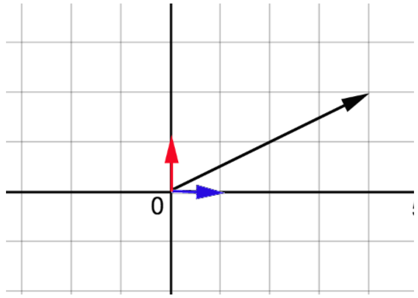
Proving Properties of Vectors:

<u>Unit Vector</u>

A vector $\vec{v}$ is called a unit vector if $\|\vec{v}\|$ = ______________

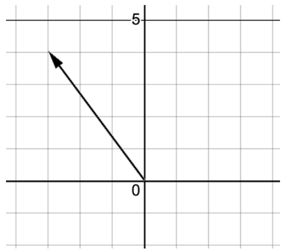
Standard basis unit vectors: $\vec{i} = \hat{i} =$ ______________ $\vec{j} = \hat{j} =$ ______________

All vectors $\vec{v} = \langle a\ b \rangle$ can be written in the form $\vec{v} = a\vec{i} + b\vec{j}$



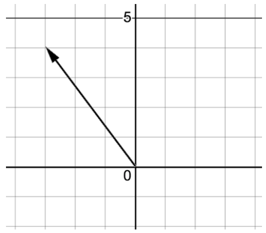
We are often interested to find unit vectors in a specified direction.

EX: Find a unit vector in the direction of $\vec{v} = <-3, 4>$



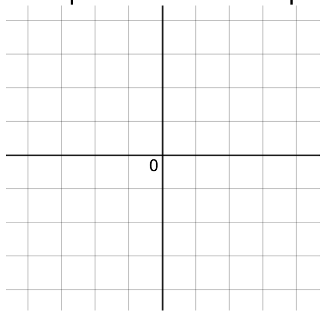
In general, a unit vector in the direction of $\vec{v}$ is given by ______________________________

EX: Find a vector of length 7 in the direction of $\vec{v} = <-3, 4>$



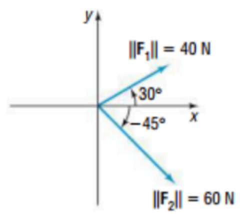
<u>Vectors can be used to show the direction of lines:</u>

Example: Find a vector parallel to the line 3x+2y=6



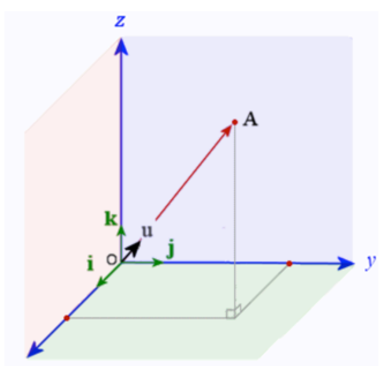
Application – Resultants of Forces (Video)

75. Resultant Force Two forces of magnitude 40 newtons (N) and 60 N act on an object at angles of 30° and −45° with the positive *x*-axis, as shown in the figure. Find the direction and magnitude of the resultant force; that is, find $\mathbf{F}_1 + \mathbf{F}_2$.



Extending to R^3 (Video)

For a vector in R^3, $\vec{v} = \langle a, b, c \rangle$ which can also be written in terms of the standard basis vectors as $\vec{v} = a\vec{i} + b\vec{j} + c\vec{k}$ where $\vec{i} = \langle 1,0,0 \rangle$, $\vec{j} = \langle 0,1,0 \rangle$, $\vec{k} = \langle 0,0,1 \rangle$. Vector computations and properties extend to R^3 as shown in the example:



Example: If $\vec{v} = \langle 1,3,-2 \rangle$ and $\vec{w} = \langle 2,5,0 \rangle$, find:

$\vec{V} + \vec{W}$=________________

$3\vec{W}$=________________

$\| \vec{V} \|$ =________________

a unit vector in the direction of $\vec{V}$________________

If P=(6, 1, 3) and Q=(-5,2,1) then $\overrightarrow{PQ}$ =______________

If not given in component form, a vector in R^3 can be described in terms of magnitude and direction angles which are the angles between the vector and each of the axes.

12.3 The Dot Product (Scalar Product, Inner Product) (Video)

1 Definition If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then the **dot product** of **a** and **b** is the number $\mathbf{a} \cdot \mathbf{b}$ given by

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Examples:

2 Properties of the Dot Product If **a**, **b**, and **c** are vectors in V_3 and c is a scalar, then

1. $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$

2. $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$

3. $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$

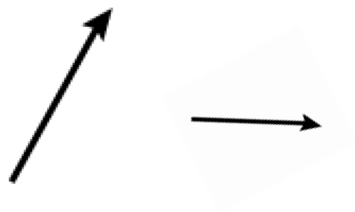
4. $(c\mathbf{a}) \cdot \mathbf{b} = c(\mathbf{a} \cdot \mathbf{b}) = \mathbf{a} \cdot (c\mathbf{b})$

5. $\mathbf{0} \cdot \mathbf{a} = 0$

Example Proof

Applications of Dot Products. (Video)

Seemingly unrelated problem: Find “the angle” between two vectors



$$\cos(\theta) = \frac{\vec{v} \bullet \vec{w}}{\|\vec{v}\| \|\vec{w}\|} \qquad \vec{v} \bullet \vec{w} = \|\vec{v}\| \|\vec{w}\| \cos(\theta) \qquad \vec{v} \bullet \vec{w} = v_1 w_1 + v_2 w_2$$

<u>Another way to find the dot product, depending on what info you are given.</u>

Example: Find **v** • **w**

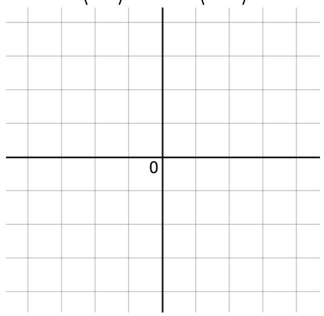
given the vectors **v** and **w** as shown, with the angle between **v** and **w** equals 60°,

llvll =5

llwll =3

Example: Find the angle between

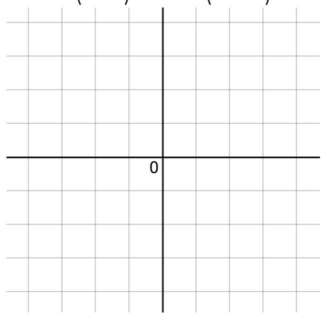
a) $\vec{v}=\langle 3,1\rangle \quad \vec{w}=\langle 2,4\rangle$



$\vec{v}\bullet\vec{w}>0$

______ $\le \theta <$ ______

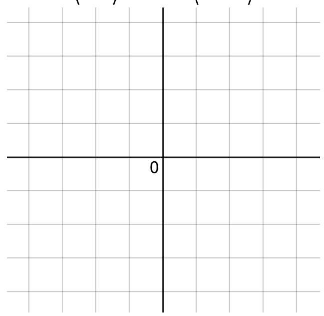
b) $\vec{v}=\langle -2,1\rangle \quad \vec{w}=\langle 3,-4\rangle$



$\vec{v}\bullet\vec{w}<0$

______ $\le \theta <$ ______

c) $\vec{v}=\langle 2,3\rangle \quad \vec{w}=\langle -3,2\rangle$



$\vec{v}\bullet\vec{w}=0$

$\theta =$ ______

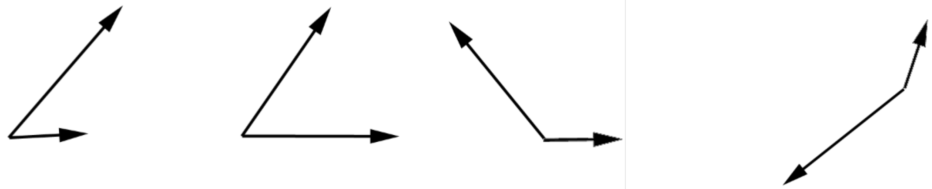
Orthogonal Vectors:

Ex: Are the vectors $\vec{v}=\langle 4,-1\rangle$ and $\vec{w}=\langle -3,2\rangle$ orthogonal?

Ex: Are the vectors $\vec{v}=\langle 7,-2\rangle$ and $\vec{w}=\langle 4,14\rangle$ orthogonal?

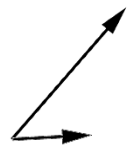
Ex: Find x such that $\vec{v}=\langle 4,x\rangle$ and $\vec{w}=\langle -5,2\rangle$ orthogonal.

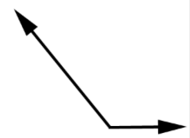
Orthogonal Projections (Video)



Derivation of the formula for finding the projection of $\vec{v}$ onto $\vec{w}$, $proj_{\vec{w}}(\vec{v})$

First notice that $proj_{\vec{w}}(\vec{v})$ is either in the direction of $\vec{w}$ or in the opposite direction, thus



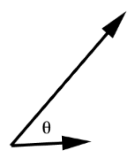


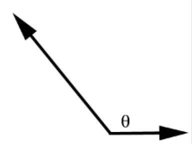
$proj_{\vec{w}}(\vec{v})=$________________ or $proj_{\vec{w}}(\vec{v})=$________________

Suppose we knew the length of the projection, call it L. (L= $\left\|proj_{\vec{w}}(\vec{v})\right\|$). Then similar to the example on page 14,

$proj_{\vec{w}}(\vec{v})=$________________ or $proj_{\vec{w}}(\vec{v})=$________________

Now let's find L





$\frac{L}{\|\vec{v}\|}=$________________ $\frac{L}{\|\vec{v}\|}=$________________

$L=$________________ $L=$________________

so as a dot product,

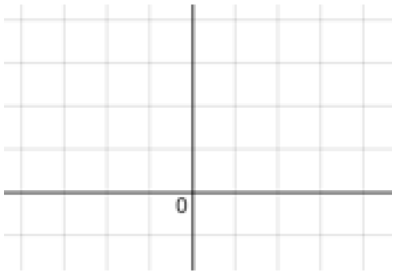
$L=$________________ $L=$________________

$proj_{\vec{w}}(\vec{v})=$________________ or $proj_{\vec{w}}(\vec{v})=$________________

Scalar projection of **b** onto **a**: $\text{comp}_{\mathbf{a}}\,\mathbf{b} = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}$

Vector projection of **b** onto **a**: $\text{proj}_{\mathbf{a}}\,\mathbf{b} = \left(\dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}\right)\dfrac{\mathbf{a}}{|\mathbf{a}|} = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\,\mathbf{a}$

Example: Let $\vec{v}=<1,3>$ and $\vec{w}=<-4,1>$ Find $proj_{\vec{w}}(\vec{v})$



<u>Example of using orthogonal projections to find distance</u>.: (class)

12.3:#53 Use projections to show that the distance from a point $P_1(x_1,y_1)$ to the line $ax+by+c=0$ is $d=\dfrac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

12.4 Cross Product (Video)

4 Definition If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then the **cross product** of $\mathbf{a}$ and $\mathbf{b}$ is the vector

$$\mathbf{a} \times \mathbf{b} = \langle a_2 b_3 - a_3 b_2,\ a_3 b_1 - a_1 b_3,\ a_1 b_2 - a_2 b_1 \rangle$$

We can use determinants to help with computation:

Matrix:

Determinant:

2X2 determinant:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

3X3 determinant:

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

Examples:

What does $\vec{a} \times \vec{b}$ look like?

<u>Direction of</u> : $\vec{a} \times \vec{b}$

In the last example, compute the following:

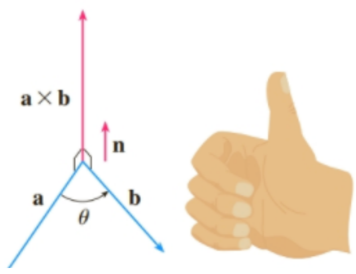
$\left(\vec{a} \times \vec{b}\right) \bullet \vec{a}$ ____________________ $\left(\vec{a} \times \vec{b}\right) \bullet \vec{b}$ ____________________

What does this tell us about the cross product?__________________________

We can prove in general, _____________________________________

With direction determined by the Right Hand Rule

SUPER HELPFUL TIP: Use this fact to check your cross products!



Length of $\vec{a}\times\vec{b}$:

9 Theorem If θ is the angle between $\mathbf{a}$ and $\mathbf{b}$ (so $0 \leq \theta \leq \pi$), then

$$|\mathbf{a}\times\mathbf{b}| = |\mathbf{a}||\mathbf{b}|\sin\theta$$

Note: if $\vec{a}\times\vec{b}=\vec{0}$ ____

(This is a good fact to know, but an easier way to determine whether vectors are parallel is ______________)

PROOF From the definitions of the cross product and length of a vector, we have

$$\begin{aligned}
|\mathbf{a}\times\mathbf{b}|^2 &= (a_2b_3 - a_3b_2)^2 + (a_3b_1 - a_1b_3)^2 + (a_1b_2 - a_2b_1)^2 \\
&= a_2^2b_3^2 - 2a_2a_3b_2b_3 + a_3^2b_2^2 + a_3^2b_1^2 - 2a_1a_3b_1b_3 + a_1^2b_3^2 \\
&\qquad + a_1^2b_2^2 - 2a_1a_2b_1b_2 + a_2^2b_1^2 \\
&= (a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2) - (a_1b_1 + a_2b_2 + a_3b_3)^2 \\
&= |\mathbf{a}|^2|\mathbf{b}|^2 - (\mathbf{a}\cdot\mathbf{b})^2 \\
&= |\mathbf{a}|^2|\mathbf{b}|^2 - |\mathbf{a}|^2|\mathbf{b}|^2\cos^2\theta \qquad \text{(by Theorem 12.3.3)} \\
&= |\mathbf{a}|^2|\mathbf{b}|^2(1 - \cos^2\theta) \\
&= |\mathbf{a}|^2|\mathbf{b}|^2\sin^2\theta
\end{aligned}$$

To help picture this length, note that if we have a parallelogram formed by vectors $\vec{a}$ and $\vec{b}$

See Geometric Illustration of Cross Product on 5C page: https://www.geogebra.org/m/RtISr7GW#material/TYfmF5Z2

11 Properties of the Cross Product If $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are vectors and c is a scalar, then

1. $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$
2. $(c\mathbf{a}) \times \mathbf{b} = c(\mathbf{a} \times \mathbf{b}) = \mathbf{a} \times (c\mathbf{b})$
3. $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$
4. $(\mathbf{a} + \mathbf{b}) \times \mathbf{c} = \mathbf{a} \times \mathbf{c} + \mathbf{b} \times \mathbf{c}$
5. $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$
6. $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$

Illustation of property 2:

Triple scalar product : not covered.

Distance from Point to Line in R3

45. (a) Let P be a point not on the line L that passes through the points Q and R. Show that the distance d from the point P to the line L is

$$d = \frac{|\mathbf{a} \times \mathbf{b}|}{|\mathbf{a}|}$$

where $\mathbf{a} = \overrightarrow{QR}$ and $\mathbf{b} = \overrightarrow{QP}$.

12.5 Equations of Lines and Planes (class)

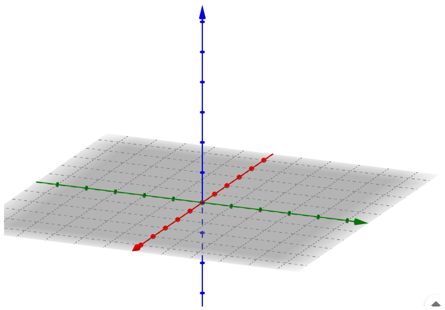
12.5i : Equations of Lines

Recall 10.1 – Parametric Equations: Given a curve in R^2, we can express it as an equation in two variables ____________________, in function form ____________________________ or as a pair of parametric equations___________________________

In R^3, an equation in 3 variables is a __________________________________. The only way to express a **curve** in R^3, is to use ____ or equivalently a vector function.

Development of Equations of Lines in R^3: What info would we need to uniquely determine a line?

Given a point on the line $P_0 = (x_0, y_0, z_0)$ and a "direction vector" $\vec{v} = \langle a, b, c \rangle$ parallel to the line



Symmetric Form:

Example: Find equations of the line through point (2, 6, 1) and parallel to $\vec{v} = \left\langle \frac{1}{2}, -3, 4 \right\rangle$

Recall: Parameterization is not ________________________________

Example: Find the equations of the line through points (3,4,-1) and (5,0, 7)

Example: Parameterizing a **line segment** : Find equations for the line *segment* from (3,4,-1) to (5,0, 7)
(for additional explanation, see the Math 5C page, "Line Segments"

Simple way to parameterize a line segment from $P_1(x_1, y_1, z_1)$ to $P_2(x_2, y_2, z_2)$:

Intersection of lines in R3: What could happen?

1)____________________________________ 3)____________________________________

2)____________________________________ 4)____________________________________

Example: Determine whether the lines intersect:

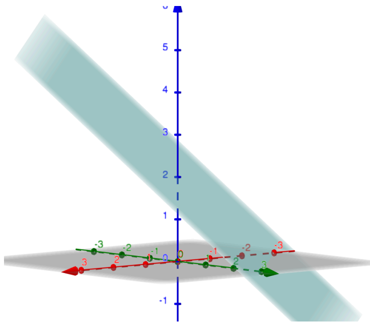
$$L_1: \begin{cases} x = 1 \\ y = 3 + 2t \\ z = 4 + t \end{cases} \qquad L_2: \begin{cases} x = -1 + 2s \\ y = 2 + s \\ z = 3 + s \end{cases}$$

Note: Be sure the lines have different parameters or you will be determining collision, not intersection

Example: Show that the lines are skew:

$$L_1: \begin{cases} x = 2t \\ y = t - 3 \\ z = 1 - t \end{cases} \qquad L_2: \begin{cases} x = s \\ y = 1 + s \\ z = 3s - 2 \end{cases}$$

12.5ii : Equations of Planes



Development of Equations of Planes in R^3: What info would we need to uniquely determine a plane?

Given a point on the line $P_0 = (x_0, y_0, z_0)$ and a "normal vector" $\vec{n} = \langle a, b, c \rangle$ orthogonal to the plane

Example: Find the equation of the plane containing point (3,1,-4) and having $\vec{n} = \langle 7, -2, 3 \rangle$

Example: Find the equation of the plane containing points ________________________________

Example: Find the equation of the plane containing lines:

$$L_1: \begin{cases} x = 1 + t \\ y = 3 - 2t \\ z = -2 + 2t \end{cases} \qquad L_2: \begin{cases} x = 2s + 4 \\ y = 2 - 4s \\ z = 4s - 1 \end{cases}$$

Intersection of 2 Planes: What could happen?

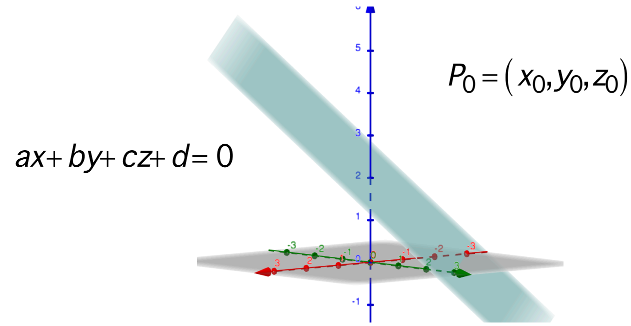
1)____________________ 2)____________________ 3)____________________

Example: Find the intersection of the planes $\begin{cases} x + y - 2z = 5 \\ 2x - y - z = 1 \end{cases}$ Two approaches

More Distance Problems: (Video)

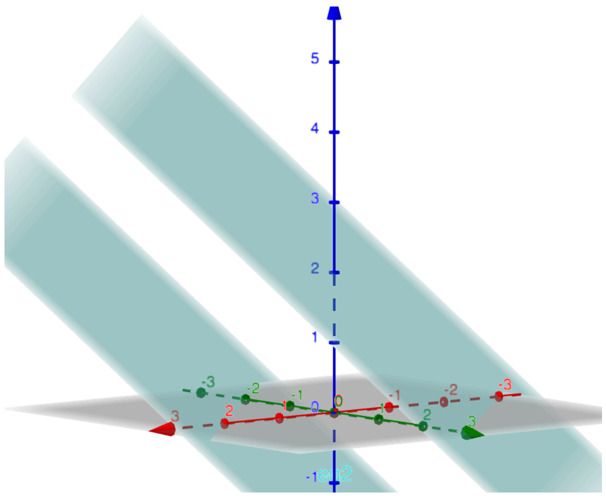
We have considered: Distance point to line in R^2 and in R^3

Distance point to plane :



Distance between parallel planes:

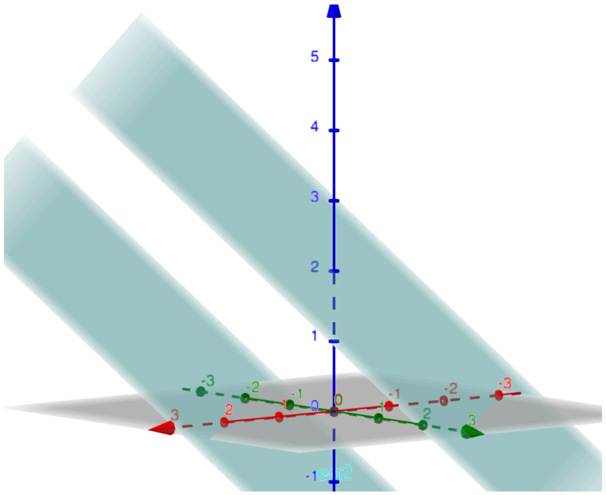
Example: Find the distance between the planes $x+2y-2z=3$ and $2x+4y-4z=7$



Distance between skew lines:

Example: Find the distance between

$$L_1: \begin{cases} x = 1 + 4t \\ y = 5 - 4t \\ z = -1 + 5t \end{cases} \qquad L_2: \begin{cases} x = 2 + 8t \\ y = 4 - 3t \\ z = 5 + t \end{cases}$$



Ans: $\frac{95}{\sqrt{1817}}$